1873MNRAS..33..375W

| A.D. | | | |
|---|---|---|---|
| 1193 | December | | |
| 1200 | August 23 | Like a date. | Seen for 6 days. |
| 1201 | January 4 | Like a date. | Seen for 12 days. |
| 1205 | February 7 | Like a date. | Seen for 13 days. |

---

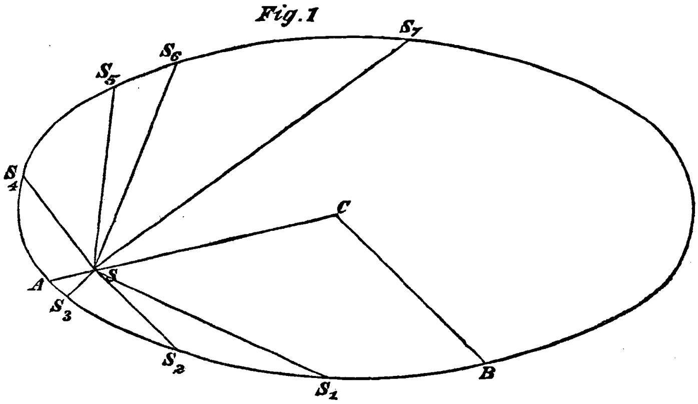
## *A Geometrical Investigation of the Orbit of a Double Star.*

By J. M. Wilson, Esq.

Having devoted spare hours occasionally to working out the orbits of double stars, I was led to endeavour to find a purely geometrical solution of the problem, "Given the orbit as projected on the plane of the sky, and the position in it of the star of reference, to find the inclination and position of the real orbit." It must be remembered, of course, that the actual orbit may be in any plane, and that the star of reference is assumed to be in its focus in that plane, but that when seen obliquely, the eccentricity of the orbit is in general changed, and the star always displaced from the focus of the projected ellipse. The problem, then, is to determine, from the amount of displacement of the focus, the position of the real ellipse. Herschel has given in the Astronomical Society's *Memoirs*, vol. v., analytical formulæ for this problem; that is, formulæ for computing the longitude of the node, the inclination of the orbit, and the angle of position of the perihelion; but, as far as I am aware, no elementary geometrical solution has been given.

Let $S_1$, $S_2$ ... $S_n$, be positions of one star with reference to another star S at different dates, the angle being got directly from observation, and corrected by the interpolating curve, and the distance being computed therefrom by Herschel's method; and let the ellipse in fig. 1 be found to pass through, or as nearly as may be through them all. Find C the centre of this, the apparent ellipse. Join C S, and produce it to meet the ellipse in

*Fig. 1*



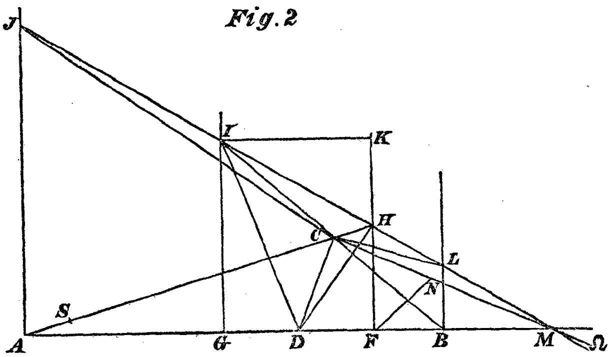
1873MNRAS..33..375W

A. This must be the projection of the axis major of the real orbit. Draw the projection of the *latus rectum*, which will be the chord that is bisected in S, and draw C B parallel to it; C B will be the projection of the minor axis of the real orbit. Also $\frac{\text{C S}}{\text{C A}}$ is unaltered by projection, and is therefore the eccentricity of the real ellipse; and therefore the ratio of the axes of the real ellipse is known. Draw C B′ at right angles to C A, and take S B′ = C A, then C A, C B′ are proportional to the semiaxes of the real ellipse.

Then the problem is evidently simply this, to draw from C two straight lines, such that their projections shall be C A and C B, such that they shall include a right angle, and such that their ratio shall be that of C A to C B′. These lines will be the semiaxes of the real orbit, and will determine its plane.

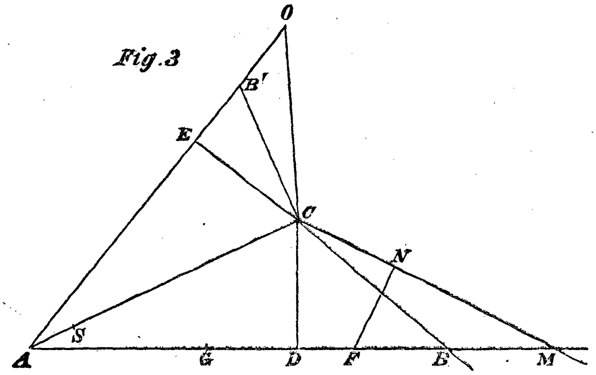
Make (in fig. 2) a perspective view of the triangle A C B.

*Fig. 2*



Draw A J, B L lines through A, B, perpendicular to the base; then the extremities of the semiaxes are to be on those lines. Draw C D perpendicular to A B (in figs. 2 and 3), and C E perpendicular to A B′.

*Fig. 3*



Divide A B in F, so that A F : F B :: A E : E B′, by joining B B′ and drawing E F parallel to B B′.

Divide A F in G, so that B F : F G :: B′ E : E C.

1873MNRAS..33..375W

Through F, G, (in fig. 2) draw lines perpendicular to the base. Determine two lines, F H, H K, such that their rectangle shall equal G F . D F, and the difference of their squares, $FH^2 - HK^2$, shall equal $GF^2 - DF^2 - CD^2$, and set off these lines F H, H K, along the line through F perpendicular to the plane A B C. Draw K I parallel to G F to meet G I in I; join H I, and produce it to meet A J, B L in J, L; and join C J, C L. Then C J, C L are the semiaxes of the real orbit.

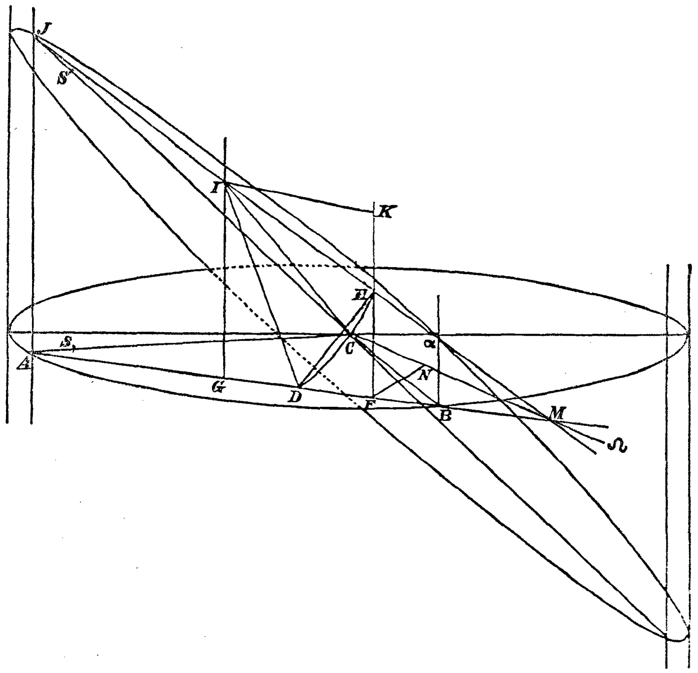
It must be shown that C J, C L, are in the required ratio C A : C B′, and that they include a right angle.

Join C I, C H, I D, D H.

Because F H, H K = D F, I K, by construction; therefore D F : F H :: H K : I K; and therefore the triangles D F H, H K I, are similar; and the angle D H F is complementary to K H I, and the angle D H I is a right angle.

And since C D is perpendicular to the plane I D H, I H is at right angles to C D, and it is also at right angles to D H, and therefore I H is at right angles to the plane C H D, and therefore I H C is a right angle.

Fig. 4.



Again, because $FH^2 - HK^2 = GF^2 - DF^2 - CD^2$, by construction, therefore $CD^2 + FH^2 + DF^2 = GF^2 + HK^2$, that is $CD^2 + DH^2 = IH^2$.

But since C D H is a right angle, $CD^2 + DH^2 = CH^2$, therefore I H = H C.

And because L H : H I :: B F : F G :: B′E : E C, therefore

1873MNRAS..33..378.

L H : H C :: B′ E : E C, and therefore the triangle L H C is similar to the triangle B′ E C.

And because L H : H J :: B F : F A :: B′E : E A and L H : H C :: B′ E : E C, therefore H J : H C :: E A : E C, and therefore the triangle C H J is similar to the triangle C E A, and the whole triangle L C J is similar to the triangle B′ C A, that is, C J : C L :: C A : C B, and L C J is a right angle. The position therefore of the orbit is found.

*To find the Line of Nodes.*—Produce J L to meet A B in M. Join C M. C M is the line of nodes, and the angle it makes with C A or C B is therefore determined (fig. 1).

*To find the Inclination.*—Draw F N perpendicular to the line of nodes. Then if $\gamma$ is the inclination, $\cot \gamma = \frac{\mathrm{NF}}{\mathrm{FH}}$ and $\gamma$ is therefore known.

*To find the Position of the Perihelion on the Orbit.*—This is the angle $\mathrm{MCJ} = 90^\circ + \mathrm{MCL}$.

Draw M O parallel to B B′ to meet A E in O; join C O. Then the angle M C L is equal to the angle O C B′.

Fig. 3 is a perspective view of the two orbits, and of the lines used in the construction.

*Temple Observatory, Rugby.*

Since the above paper was read I have received No. 1227 of the *Astronomische Nachrichten*, and find that it contains a geometrical solution by Thiele of the same problem. Thiele's solution is very much simpler and better, and mine is nothing therefore but a geometrical exercise, which has the advantage of exhibiting the two orbits in their connexion with one another, and its method may have useful application in solid geometry, and so I let it stand.

---

## *American Preparations and Stations for the Observation of the Transit of Venus of* 1874.

(*Extract of a Letter from Rear-Admiral B. F. Sands to the Astronomer Royal.*)

In answer to your letter of February 27, I have much pleasure in making known to you our plans for the observation of the Transit of *Venus*, so far as they have been matured.

As to the mode of observing, our main reliance will be upon photographs of the Sun, with *Venus* on his disk, taken on the plan described by Professor Newcomb in Part I. of the "Papers relating to the Transit of *Venus*." In applying this plan, it will be admissible to choose stations where the Sun shall be 10° or more above the horizon during the entire period of the transit, and